

# Drying with superheated steam: maximum drying rate as a linear function of pressure

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## Abstract

Drying with superheated steam (SS) as heating agent is controlled through three operative variables, the steam temperature, the recycle velocity, and the system pressure. Once the other conditions are fixed, there is an optimum pressure at which drying rate reaches its maximum value. The exact optimum conditions can be found through a differential mass and energy balance over the heat exchange area, resulting in two equations. The first one to calculate the drying rate as function of the operating conditions and the second one to find the values of those conditions which make the operation optimum. Then, using these equations, this paper shows that the relationship between maximum drying rate and pressure can be rewritten in a unique linear equation. Experiments were performed in a model system to obtain a family of drying rates versus pressure curves, and in accordance with the predicted behaviour, it was observed that all maximum drying rates lay on a straight line. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Optimum pressure; Drying rate; Superheated steam

## 1. Introduction

It has been experimentally observed and also formally proved [1] that the drying rate with superheated steam (SS) always increases with steam temperature and recycle velocity, but for these two parameters given, there is an optimum pressure at which drying rate reaches its maximum value. This maximum occurs between two pressures for which drying rate becomes zero. The first value is zero pressure, because in this case the SS density is also zero. The second value is the pressure at which the sample equilibrium temperature is equal to the SS temperature, because in this case the thermal driving force is null.

The exact optimum conditions also depend on the characteristics of the drier, and in order to find them a differential mass and energy balance must be solved over the heat exchange area. The mathematical derivation results in two equations, one to calculate the drying rate as a function of the operating conditions and the other one to find the optimum set of those conditions, such that the maximum drying rate can be found by combining these two equations. However, though the method is mathematically exact, the optimum condition equation is not explicit and the procedure is arduous.

In this paper, it is shown that for all other conditions given, the maximum drying rate is a linear function of the

pressure, that is having defined the steam velocity and temperature, the value of pressure which makes the evaporating rate maximum is easily calculated.

## 2. Theory

Fig. 1 shows a schematic diagram of a SS drying process. A certain mass,  $F$  of SS flows over the wet sample transferring part of its sensible heat,  $Q$ . This amount of energy produces evaporation at a rate of  $m \text{ kg s}^{-1}$  of water that leaves the sample as vapour and joins the SS main flow. To maintain constant the process conditions, SS is continuously recycled so that heat is replaced and vapour produced by evaporation is withdrawn.

Heat is transferred from the SS to the sample under the driving force imposed by the difference between the SS temperature,  $T_{SS}$ , and the sample equilibrium temperature,  $T^{eq}$ , and can be expressed by the well known transfer equation:

$$Q = AU(T_{SS} - T^{eq}) \quad (1)$$

where  $A$  is the sample external area and  $U$  the overall heat transfer coefficient.

Drying rate is a direct function of the heat transferred upon the water latent heat of vaporisation,  $\Delta H$ :

$$m = \frac{Q}{\Delta H} \quad (2)$$

The value of  $Q$  as given by Eq. (1) changes along the drying because SS temperature decreases as it crosses the drying

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**Nomenclature**

$A$	sample external area ( $\text{m}^2$ )
$C_p$	mean SS heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
$C_p^v$	mean removed vapour heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
$F$	superheated steam flow rate ( $\text{kg s}^{-1}$ )
$h$	external heat transfer coefficient ( $\text{J m}^{-2} \text{K}^{-1}$ )
$H$	enthalpy ( $\text{J kg}^{-1}$ )
$\Delta H$	vapour latent heat of vaporisation ( $\text{J kg}^{-1}$ )
$K$	factor in Eq. (18) ( $\text{s}^2 \text{m}^{-2}$ )
$m$	drying rate ( $\text{kg s}^{-1}$ )
$N$	relative variation of $h$ with pressure
$Nq$	dimensionless number
$P$	pressure ( $\text{Pa} = \text{N m}^{-2}$ )
$Q$	heat (W)
$R$	universal gas constant ( $\text{J mol}^{-2} \text{K}^{-1}$ )
$\mathfrak{R}_{Nq}$	dimensionless function
$S$	volume normal section ( $\text{m}^{-2}$ )
SS	acronym for superheated steam
$T$	temperature (K)
$U$	overall heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ )
$v$	linear velocity ( $\text{m s}^{-1}$ )
$W$	water molecular weight ( $\text{kg mol}^{-1}$ )

*Greek symbol*

$\varphi$	heat transfer coefficient factor
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*Subscripts*

i	inlet
m	mean value
max	maximum value
o	outlet
op	optimum value
SS	superheated steam

*Superscript*

eq	equilibrium value
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section and also the local heat transfer coefficient changes with geometry and sample conditions. In order to apply this equation, a mass and energy balance was written over a differential control volume as it is shown in Fig. 2. This control volume includes the outer portion of the sample already dried at a given instant of the process, in which hygroscopic equilibrium has been reached and no further evaporation occurs. This has been named the “dry layer”.

The heat released by the SS as it flows along the volume length causes its temperature to decrease such that  $T_i > T_o$ , and correspondingly mass flow rate increases due to the water vapour released by the sample, such that  $F_o > F_i$ .

Steady state is assumed (SS temperature varies very slowly with time so any change can be neglected), and for

the purpose of this study it is assumed adiabatic behaviour (drying zone is perfectly isolated). Now, neglecting SS potential and kinetic energy changes, macroscopic mass and energy balances for non-isothermal flow system are the following [2]:

$$F_i + m = F_o \quad (3)$$

$$F_i H_i + m H^{\text{eq}} = F_o H_o + Q \quad (4)$$

where  $H_i$  and  $H_o$  are the SS local enthalpies and  $H^{\text{eq}}$  is the vapour enthalpy at  $T^{\text{eq}}$ , the equilibrium temperature of the wet sample.

Superheated steam pressure is assumed approximately constant through all drying zone. Under such a condition, the difference between inlet and outlet SS enthalpies can be calculated as a mean value of SS heat capacity ( $C_p$ ) multiplied by the difference between inlet and outlet SS temperature [3]:

$$H_i - H_o = C_p (T_i - T_o) \quad (5)$$

In order to combine mass and energy balances in one equation,  $F_o$  from Eq. (3) is replaced into Eq. (4), SS enthalpy difference is replaced by Eq. (5) (a similar relation is used for the difference on removed vapour enthalpy), and  $m$  is related to  $Q$  through Eq. (2):

$$F_i C_p (T_i - T_o) = Q \left[ \frac{1 + C_p^v (T_o - T^{\text{eq}})}{\Delta H} \right] \quad (6)$$

where  $C_p^v$  is the mean value of water vapour heat capacity.

Using the same procedure, mass and energy balance is recalculated using  $F_o$  instead of  $F_i$ , and dividing both balances the relation between inlet and outlet SS mass flows is obtained as follows:

$$\frac{F_o}{F_i} = \frac{1 + C_p^v (T_i - T^{\text{eq}}) / \Delta H}{1 + C_p^v (T_o - T^{\text{eq}}) / \Delta H} \approx 1 \quad (7)$$

For most practical purposes, it is  $C_p^v (T_{\text{SS}} - T^{\text{eq}}) \gg \Delta H$ , hence it can be assumed that SS mass flow is approximately constant, i.e.  $F_i \approx F_o \approx F$ .

Now, Eqs. (1) and (6) are applied to a differential area (such that the temperature difference is replaced by a temperature differential ( $T_o - T_i = -dT_{\text{SS}}$ ), and total heat and area become differential heat ( $Q = dQ$ ) and area ( $A = dA$ ), and then combined. The resulting differential equation is integrated over the entire exchange surface, and assuming  $F \approx$  constant and neglecting the dependence of heat capacity on temperature, the following equation results:

$$\frac{AU_m}{FC_p} = \ln \left( \frac{1 + C_p^v (T_o - T^{\text{eq}}) / \Delta H}{1 + C_p^v (T_i - T^{\text{eq}}) / \Delta H} \frac{T_i - T^{\text{eq}}}{T_o - T^{\text{eq}}} \right) \quad (8)$$

where  $U_m$  is the mean overall heat transfer coefficient.

Finally, combining Eqs. (2), (6) and (8), Eq. (9) is obtained, which shows that drying rate  $m$  is function of  $F$

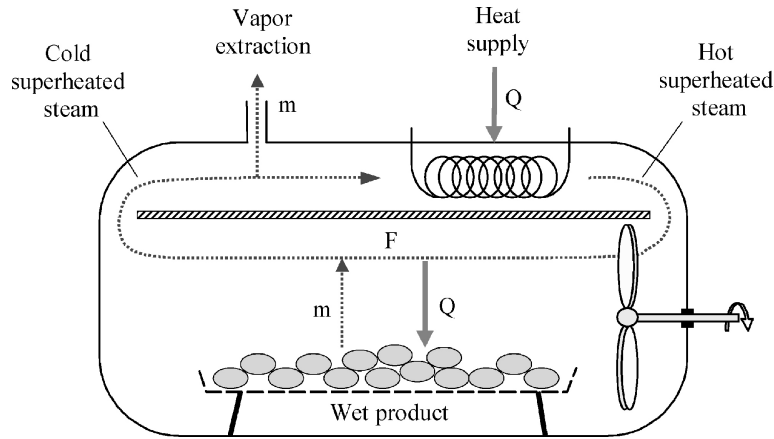


Fig. 1. Schematic diagram of superheated steam drying process.

(constant),  $T^{eq}$ ,  $\Delta H$ ,  $C_p$ ,  $T_i$ , and a dimensionless number  $Nq$ , related to the cooling effects on the SS:

$$m = \frac{FC_p(T_i - T^{eq})}{\Delta H} (1 - e^{-Nq}) \quad (9)$$

Factor  $Nq$  relates the amount of heat transferred by convection to the amount of heat transported by the SS as sensible heat, and it is defined in Eq. (10):

$$Nq = \frac{AU_m}{FC_p} \quad (10)$$

And assuming that  $C_p^v(T_{SS} - T^{eq}) \gg \Delta H$ , Eqs. (8) and (10) lead to show that the SS temperatures and  $Nq$  number are related by means of Eq. (11):

$$Nq \approx \ln \left( \frac{T_i - T^{eq}}{T_o - T^{eq}} \right) \quad (11)$$

Ideal gas behaviour can be assumed at the prevailing pressure. Hence, the SS flow,  $F$  can be calculated by means of Eq. (12):

$$F = S \frac{W v P_{SS}}{R T_{SS}} \quad (12)$$

where  $S$  is the drier normal section;  $W$  the water molecular weight;  $P_{SS}$ ,  $v$ , and  $T_{SS}$  are the SS pressure, linear velocity, and temperature, respectively;  $R$  is the universal gas constant.

From Eqs. (9) and (12), it follows that there are two pressure values at which the drying rate is zero. The first one is  $P_{SS} = 0$ , because it makes  $F = 0$  in Eq. (12), and no operation takes place. The second value is that which makes  $T^{eq} = T_i$ . Between those two points must be at least one pressure at which the drying rate reaches a maximum.

Elustondo [1] derived expression (13) which represents the condition at which maximum drying rate is obtained. For all operating parameters fixed, including  $T_i$ ,  $T^{eq}$  is defined, and in turn  $P_{SS}$  is defined representing the optimal SS pressure:

$$T_i \approx \frac{R}{W \Delta H} \left[ \frac{(T^{eq})^2}{1 - \mathfrak{R}_{Nq}(1 - \varphi N)} \right] + T^{eq} \quad (13)$$

In this condition, there are two new dimensionless parameters ( $\varphi$  and  $N$ ) and a new dimensionless function ( $\mathfrak{R}_{Nq}$ ). The first parameter ( $\varphi$ ) is a relation between the mean overall heat transfer coefficient  $U_m$ , and the mean value surface heat transfer coefficient  $h_m$ :

$$\varphi = \frac{U_m}{h_m} \quad (14)$$

The second parameter ( $N$ ) is the relative variation of  $h_m$  with pressure:

$$N = \frac{P_{SS}}{h_m} \frac{\partial h_m}{\partial P_{SS}} \quad (15)$$

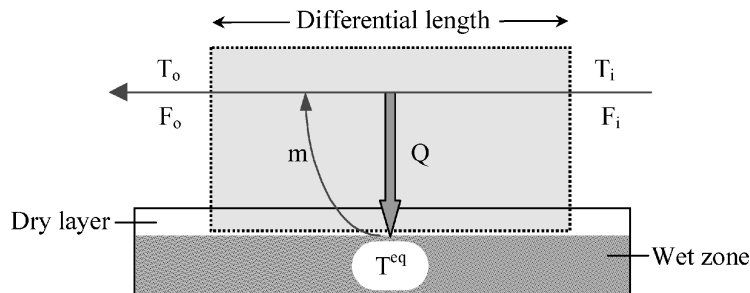


Fig. 2. Heat and vapour transfer in a differential portion of the system.

Finally, the new function ( $\mathfrak{R}_{Nq}$ ) is exclusively a function of the dimensionless number  $Nq$ :

$$\mathfrak{R}_{Nq} = \frac{Nq e^{-Nq}}{1 - e^{-Nq}} \quad (16)$$

In order to calculate the maximum drying rate, expressed as the maximum vapour evolved from the sample per unit time,  $m_{\max}$ , condition (13) is replaced into Eq. (9), and Eq. (12) is applied at the drying zone entrance. The result shows that the maximum drying rate is directly proportional to the inlet sectional area,  $S_i$ , the inlet recycle velocity,  $v_i$ , and the SS pressure which becomes optimum,  $P_{\text{op}}$ :

$$m_{\max} = K S_i v_i P_{\text{op}} \quad (17)$$

where the proportion factor  $K$  is calculated by means of Eq. (18):

$$K = \frac{C_p}{(\Delta H)^2} \frac{(T^{\text{eq}})^2}{T_i} \frac{1 - e^{-Nq}}{1 - \mathfrak{R}_{Nq}(1 - \phi N)} \quad (18)$$

This factor is almost independent of temperature, for relation  $(T^{\text{eq}})^2/T_i$  is balanced since  $T_i$  in Eq. (13) is a quadratic function of  $T^{\text{eq}}$  and  $T$ 's are absolute. On the other hand, while  $Nq$  varies from 0 to  $\infty$ , factor  $e^{-Nq}$  and function  $\mathfrak{R}_{Nq}$  only vary from 1 to 0, hence  $K$  could be expected to be approximately constant under a large number of operating conditions.

### 3. Materials and methods

Fig. 3 shows the experimental set up. A centrifugal blower recycles the SS through the drying zone and past the heating electrical resistance. The drying zone is a cylindrical tube of 0.3 m diameter and 0.4 m in length closed in its backside, and with a glass lid covering the front side allowing the inspection of samples as drying proceeds. Low pressure is maintained with a vacuum pump, and an inlet purge steam is used to supply vapour when process starts. The steam recycle velocity is controlled through the blower rotational speed, and the inlet temperature is measured in the drying zone inlet tube and controlled by adjusting the heat supplied by the electrical resistance.

This drier operates at approximately constant temperature, so in order to simulate the cooling effects an auxiliary device was adapted (see Fig. 4). It consists of a cylindrical insulating tube of 0.078 m i.d. placed into the drying zone just over the drying zone inlet tube. Inside the tube there is an aluminium can of 0.067 m diameter and 0.070 m height filled with distilled water and supported by a metallic mesh. The lower part of the can is isolated to control the evaporation and to prevent water spatter. There are only three experimental measurements, temperatures  $T_i$ ,  $T_o$  and  $T^{\text{eq}}$ , made with three thermocouples, one in the drying zones inlet tube, another inside the can's water, and the third one in the upper part where the cold superheated steam leaves the device.

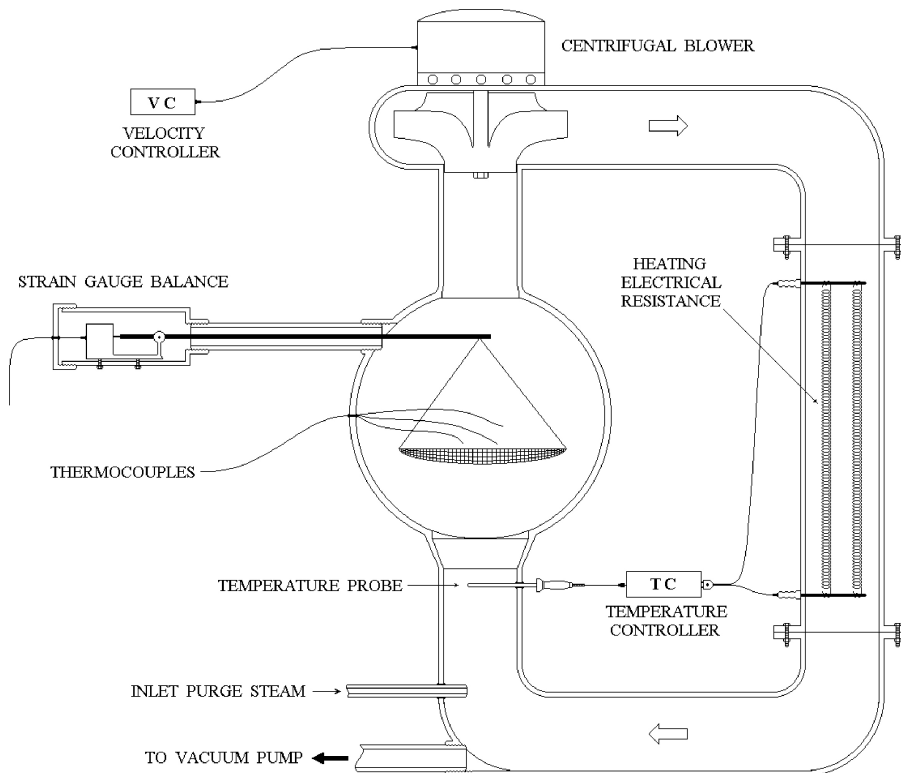


Fig. 3. Experimental superheated steam drying equipment.

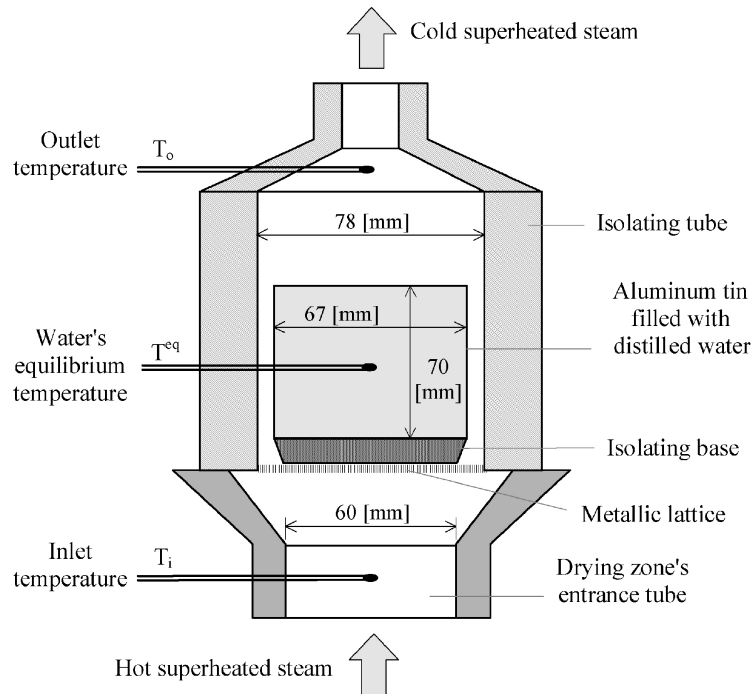


Fig. 4. Experimental device to simulate the cooling effect.

The can wall has negligible thermal resistance, hence  $U_m \approx h_m$  which makes parameter  $\varphi$  in Eq. (14) close to unity. In drying foodstuffs the evaporated water gives place to a growing dry layer, increasing the thermal resistance which means decreasing  $U_m$ . But for the initial stages of the process, the situation is similar to that represented by the can, so it can be assumed that  $\varphi = 1$  and results are applicable to any substance with the same shape.

The influence of radiant-heat transfer was calculated in accordance to the Stefan–Boltzmann law, assuming the walls are at the SS temperature, and the sample is at  $T^{eq}$ . Results show that radiant contribution is negligible, in the order of 2% of the convective component. On the other hand, since the overall heat transfer coefficient was experimentally measured, the resulting parameter involves both contributions (Kreith [4]), and the calculated evaporation rates are well reproduced experimentally.

The experimental measurements were made at constant inlet temperatures of  $T_i \approx 335, 343, 353,$  and  $363$  K, constant inlet recycle velocity  $v_i = 1.74 \text{ m s}^{-1}$ , and pressure varying from  $P_{SS} = 1 \times 10^4$  to  $3 \times 10^4$  Pa. Results show that experimental difference between inlet and outlet temperature increase when inlet temperature increase, varying from 6 to 22 K at  $1 \times 10^4$  Pa and from 0 to 7 K at  $3 \times 10^4$  Pa. Dimensionless number  $Nq$  is calculated by Eqs. (8) and (10), using the experimental temperatures and reading  $C_p^v$  and  $\Delta H$  from thermodynamic steam tables. Results show that  $Nq$  increases with inlet temperature and decreases with pressure, varying from a minimum  $Nq = 0.296$  (335 K and  $2 \times 10^4$  Pa) to a maximum  $Nq = 0.636$  (363 K and  $1 \times 10^4$  Pa). Finally, using non-linear fitting, the experimental  $Nq$  values are represented

by empirical Eq. (19):

$$Nq = \frac{1}{430} \frac{T_i - 250}{F^{0.453}} \quad (19)$$

Since the can external surface has almost no thermal resistance, the overall heat transfer coefficient is the same as the external heat transfer coefficient ( $U_m = h_m$ ), so that Eq. (19) can be used in combination with Eq. (10) to find  $h_m$ , and then this information is applied in Eq. (15) to obtain parameter  $N$ :

$$N = 0.547 \quad (20)$$

This empirical datum, together with the SS heat capacity and values of latent heat from steam tables, allow the calculation of both the drying rate velocity  $m$  (Eq. (9)), and the optimum drying conditions  $P_{op}$  (Eq. (17)). Later, optimum conditions are used to calculate parameter  $K$  (Eq. (18)). The calculated  $K$  values do not change significantly, and under our conditions it can be approximated using its mean value  $K = 6.4 \times 10^{-8} \text{ s}^2 \text{ m}^{-2}$ . Then,  $K$  being a constant, Eq. (17) can be rewritten to allow the calculation of the maximum drying rate without the previous knowledge of the optimum operating conditions:

$$m_{\max} = 6.4 \times 10^{-8} S_i v_i P_{SS} \quad (21)$$

Even though this equation is only applicable to the particular case of our experimental set up, it confirms the assumption that parameter  $K$  can be assumed constant and, consequently, that relationship between pressure and optimum drying rate is linear.

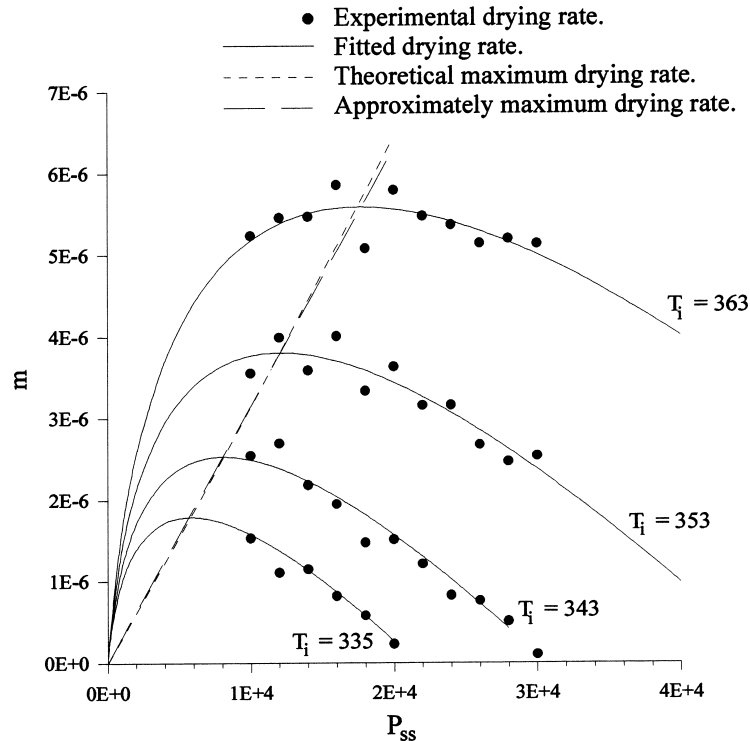


Fig. 5. Theoretical and approximated maximum drying rates as a function of working SS pressures.

Finally, Fig. 5 compares the theoretical and approximated maximum drying rates. Black dots are the drying rates calculated with Eq. (9) using the experimental data, while curves are the drying rates calculated with same Eq. (9), using constant  $T_i$  and the empirical Eq. (19). The dotted and dashed lines show the maximum drying rate as calculated with theoretical Eqs. (17) and (18), and with the linear Eq. (21), respectively.

#### 4. Conclusion

Drying with superheated steam has the characteristic that, for all other conditions defined, does exist an optimum working pressure at which the drying rate is maximum. The optimum conditions can be determined analytically by a somehow tedious procedure. However, it has been shown that the problem can be simplified assuming a linear relationship, which can be easily determined.

The theoretically derived linear relationship is shown as Eq. (17), but since parameter  $K$  is expected to be relatively

constant, it can be calculated at a given intermediate point and then used in a wide range of conditions. Then, Eq. (21) is obtained, which is independent of the SS temperature and can be applied without previous knowledge of optimum conditions.

The main advantage of the latter procedure is that the straight line turns to be the locus of the maxima shown by each drying rate isotherm, hence the linear relationship between maximum drying rate and pressure is an easy and fast way to estimate the optimum performance of the drier.

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